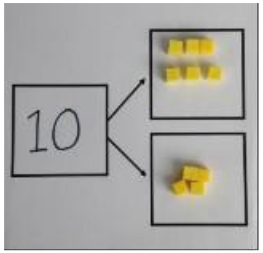
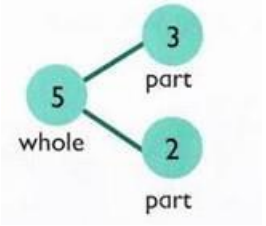
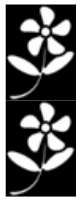
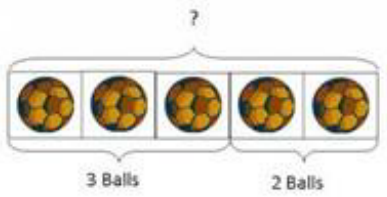
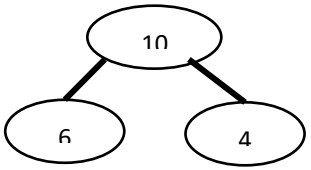
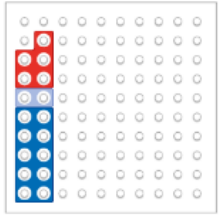
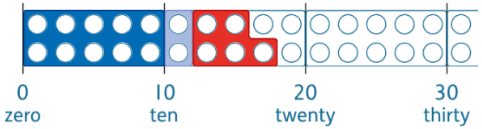
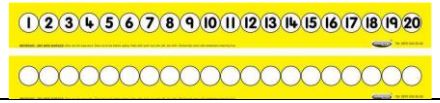
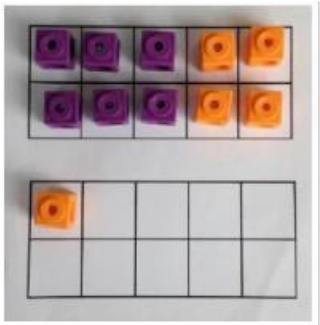
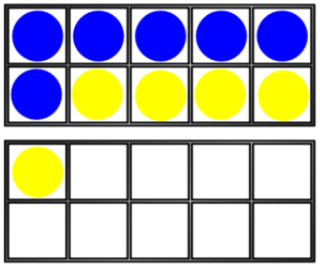
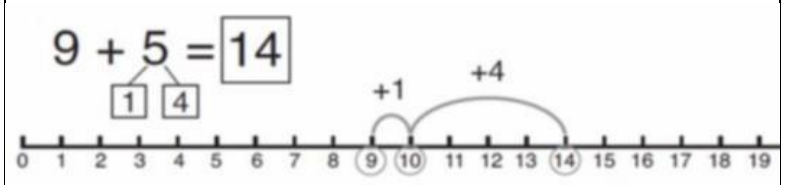
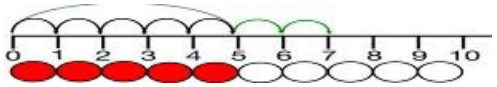
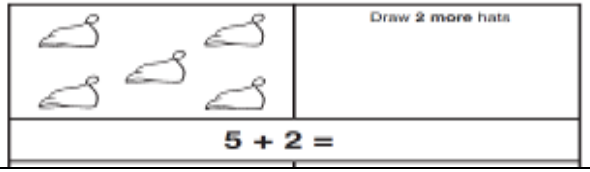
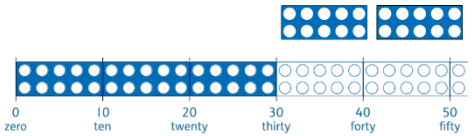
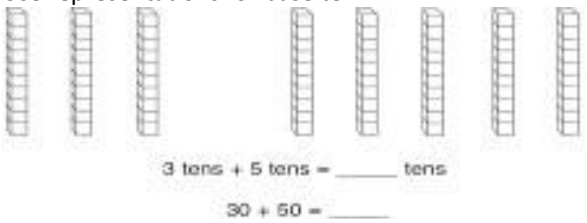
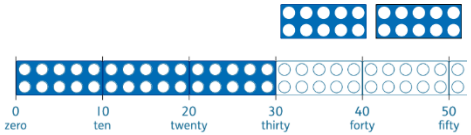
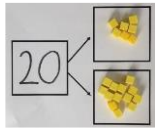
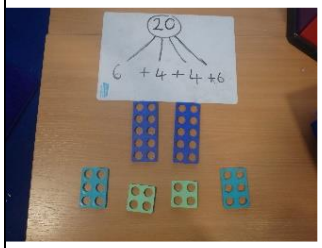
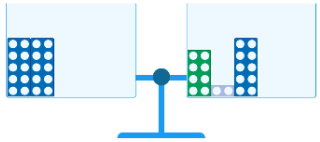
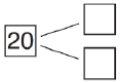
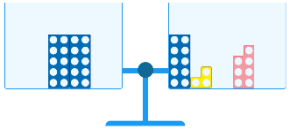
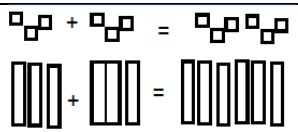
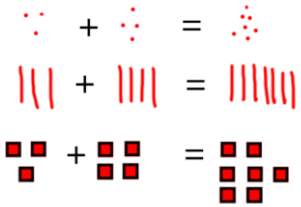
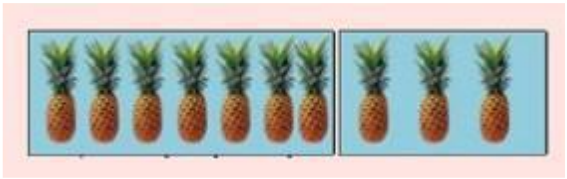
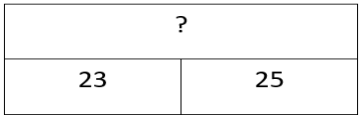
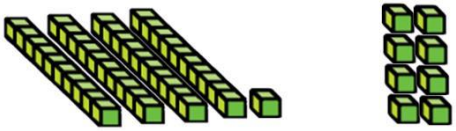
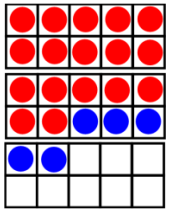
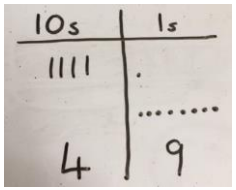
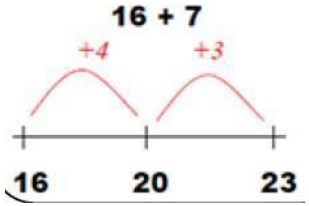
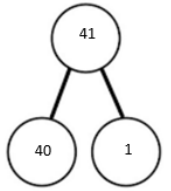
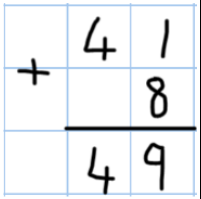
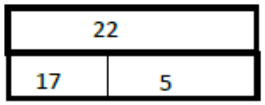
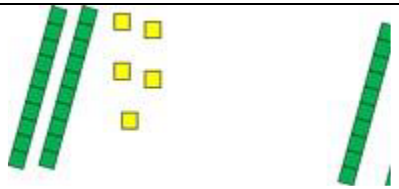
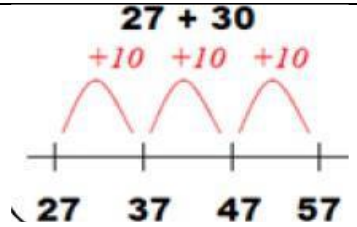
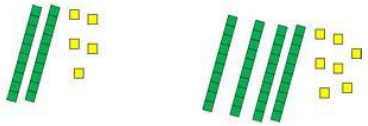
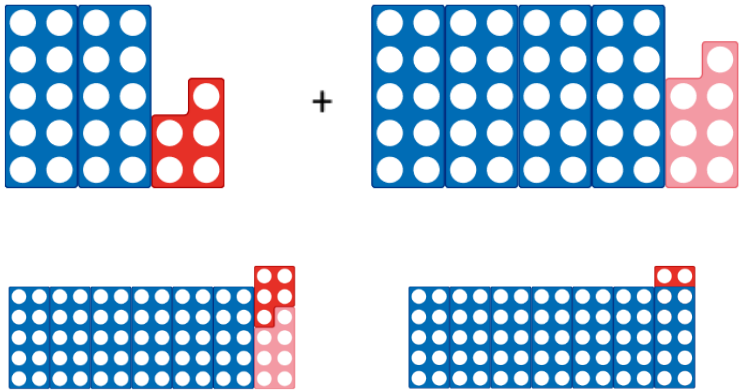
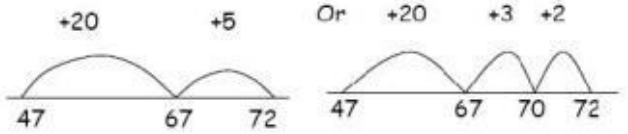
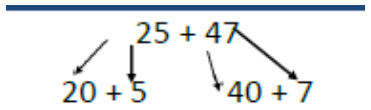
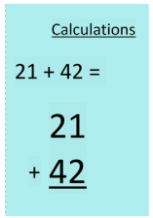
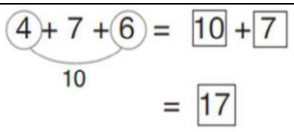
Year 1 Addition			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Combining two parts to make a whole: part whole model.	   Use part part whole model. Use cubes to add two numbers together as a group or in a bar. 	   3 Balls 2 Balls Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7  10 = 6 + 4  Use the part-part whole diagram as shown above to move into the abstract It's important children experience the = sign in different places in the calculation.
Starting at the bigger number and counting on	 12+ 5 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. Use Numicon Shapes 	12 + 5 = 17 Start at the larger number on the number line and count on in ones or in one jump to find the answer.   Use numicon shapes on a number line (numicon IWB)	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

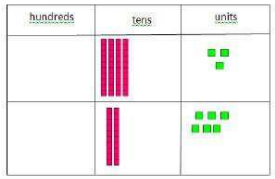
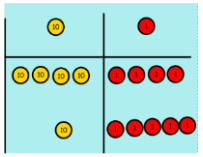
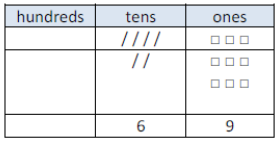
	Use counters on a number track to count on. 		
Regrouping to make 10. *This is an essential skill for column addition later.*	6 + 5 = 11   Start with the bigger number and use the smaller number to make 10. **Use ten frames.**	Use a tens frame or a number line. Children to draw the ten frame and counters/cubes  Regroup or partition the smaller number using the part part whole model to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now? Children to develop an understanding of equality e.g. 6 + □ = 11 6 + 5 = 5 + □ 6 + 5 = □ + 4
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.	 	Emphasis should be on the language ‘1 more than 5 is equal to 6.’ *‘2 more than 5 is 7.’* *‘8 is 3 more than 5.’*

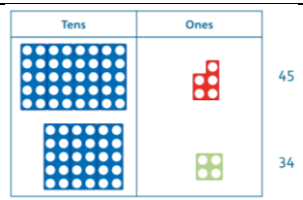
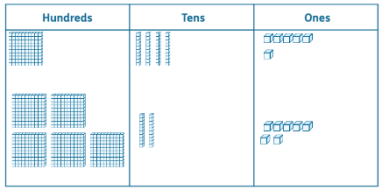
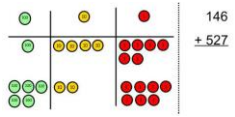
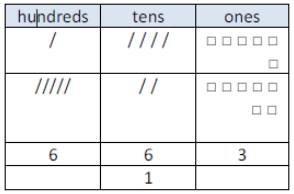
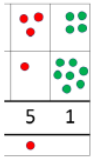
Year 2 Addition			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Adding multiples of ten	50= 30+20  Model using dienes and bead strings Use Numicon shapes on a number line  	Use representations for base ten.  Use Numicon shapes on a number line (IWB)  	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part-part whole	 Children explore ways of making numbers within 20 (cubes, objects, numicon)  Use balancing scales to find ways of making numbers within 20 with the Numicon shapes. 	 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □ Use balancing scales to find ways of making numbers within 20 with the Numicon shapes (IWB)  20 = 13 + 7.	□ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1

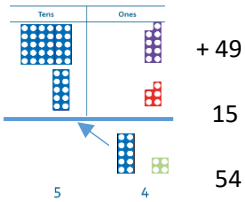
Using known facts	 Use Dienes or Numicon shapes. 3 ones + 3 ones = 6 ones = 6 3 tens + 3 tens = 6 tens = 60.		3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700
Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48
Add a two-digit number and ones	  17 + 5 = 22 Use ten frame to make 'magic ten' Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.  	41 + 8  1 + 8 = 9 40 + 9 = 49  If children are ready model column addition alongside these methods. Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 

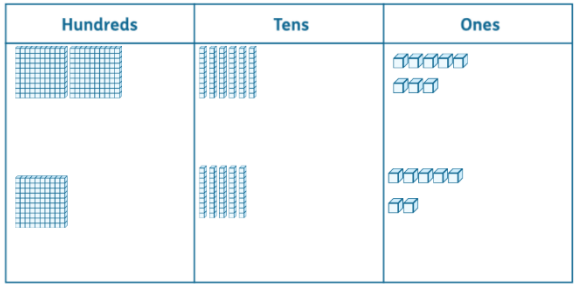
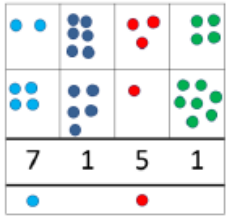
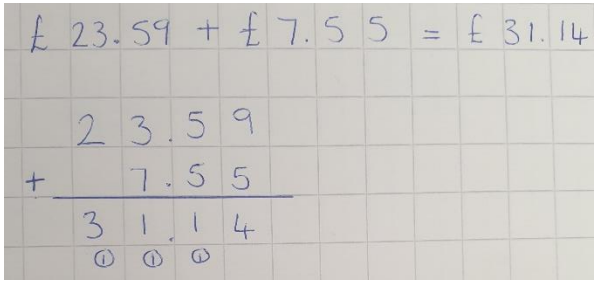
Add a 2 digit-number and tens	 25 + 10 = 35 Explore that the ones digit does not change	 Children can use a number line to count in steps of ten, reinforcing the ones digit does not change.	27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Add two 2-digit numbers	 Model using dienes, place value counters and Numicon.  = 72	 Use number line and bridge ten using part whole if necessary, starting at the largest number.	 20 + 40 = 60 5+ 7 =12 60 + 12 = 72 *If children are ready introduce column method – keep numbers friendly*  Non statutory guidance from NC *'Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.'*

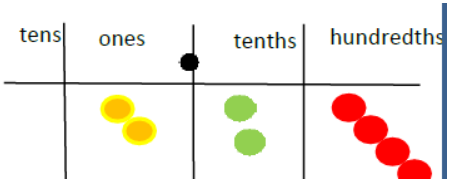
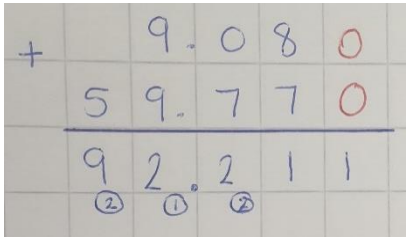
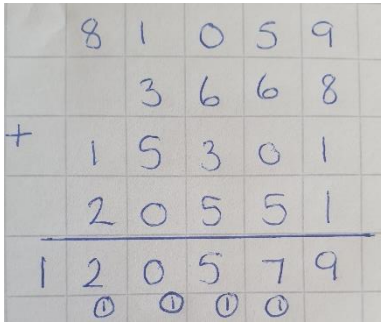
Add three 1-digit numbers	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Encourage the children to use known facts. Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.  The 4 and the 6 can be grouped together to make 10. Then add on the 7.	Add together three groups of objects. Draw a picture to recombine the groups to make 10.  4 + 6 + 7 = 17	 Combine the two numbers that make 10 and then add on the remainder.

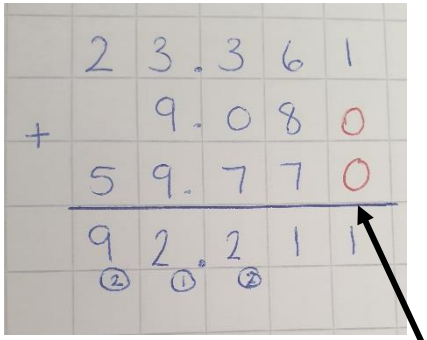
Year 3 Addition			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3- digit numbers.	 43 + 26 Use Dienes to add tens and ones before moving on to place value counters. Modelling the recording of the calculation alongside (building from Yr2)  Numicon can be used to show the tens and ones.	After practically using the base 10 blocks and place value counters, children can draw the Dienes to help them to solve addition calculations.  Children move to drawing the counters using a tens and one frame. 	Calculations 21 + 42 = 21 + 42 Only select numbers which do not involve regrouping. 2 2 3 + 1 1 4 Add the ones first, then the tens, then the hundreds.

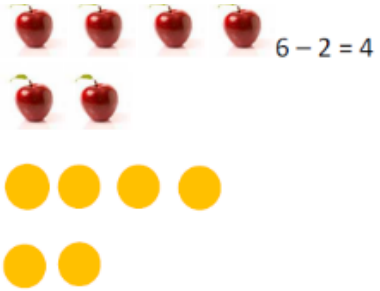
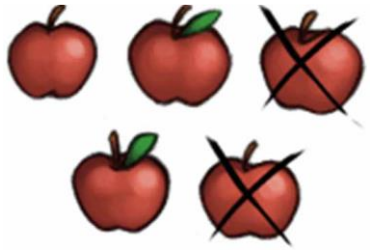
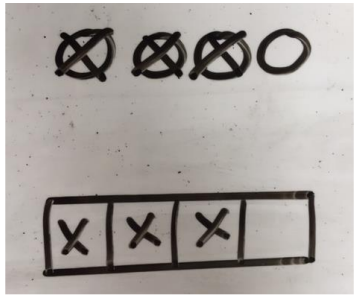
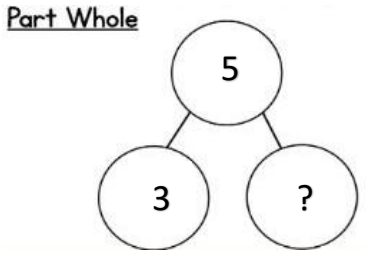
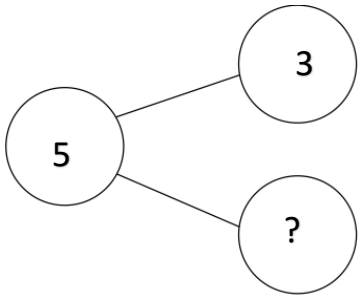
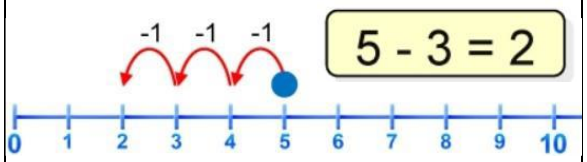
	 **Place value arrow cards.** Partition each number (Use place value cards to help partition, move the 10s together, move the 1s together), 2. Add the 10s and 1s 3. Recombine the answers to find the total 65 + 24 = 60 + 20 + 5 + 4 = 80 + 9 = 89		
Column method regrouping	Add up the units and exchange 10 ones for one 10 and so on.  146 + 527 This helps children clearly see that 10 ones equal 1 ten and 10 tens equal 100. Continue using place value counters as children begin to work with decimals. 	 146 +527  Children can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line	20 + 5 40 + 8 60 + 13 = 73 Start by partitioning the numbers before formal column to show the exchange. 536 + 85 621 1 1 Carry digits are recorded underneath, using the words ‘carry one ten’ or ‘carry one hundred’, **not** ‘carry one’.

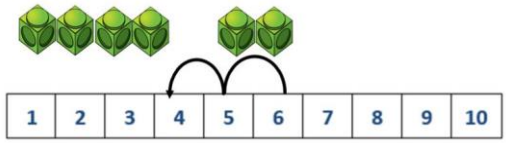
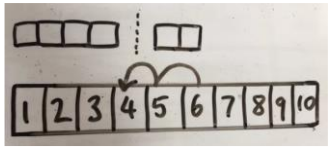
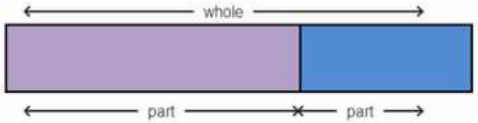
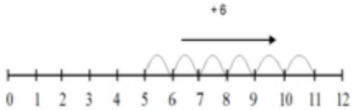
	 + 49 15 54		
	We want children to be able to make rich connections between numbers and choose appropriate methods for calculating. This may be a written method but also may be a mental method.		

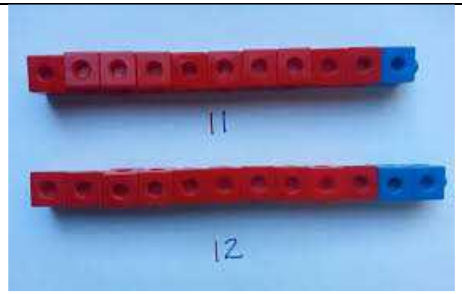
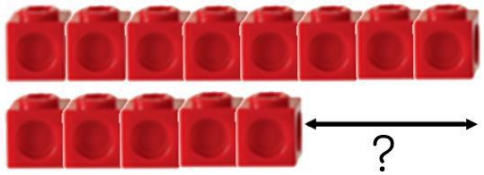
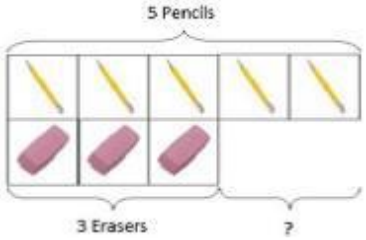
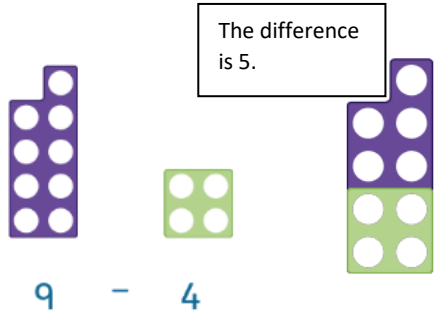
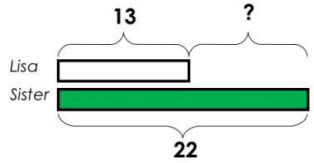
Year 4 – 6 Addition			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Y4—add numbers with up to 4 digits Add decimals to two decimal places, including money and measurement.	Children continue to use dienes or PV counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using place value grid	Continue from previous work to carry hundreds as well as tens. Relate to money and measures. 

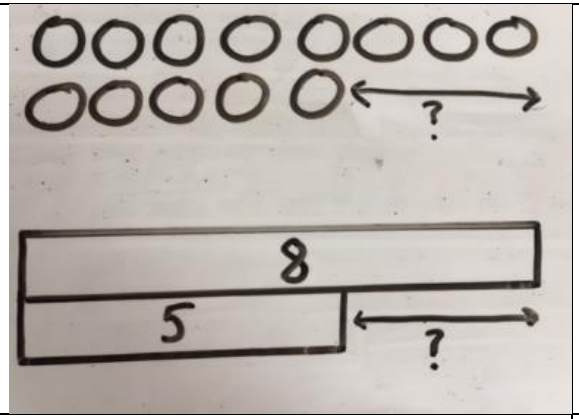
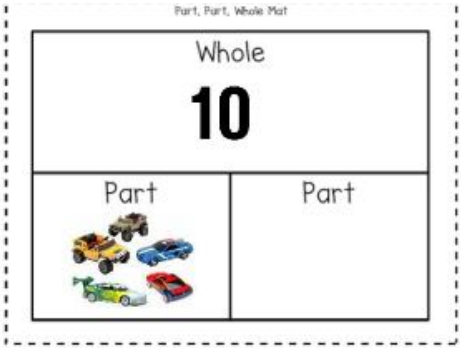
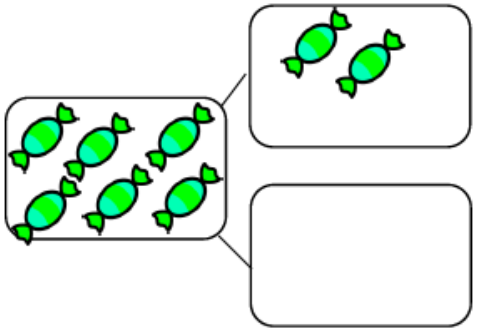
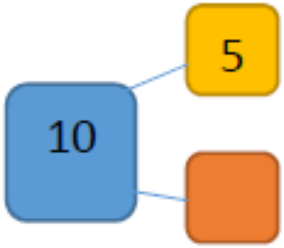
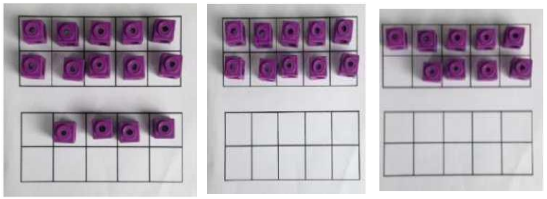
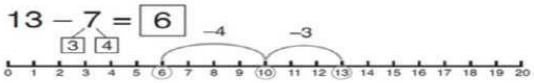
Y5 - Add numbers with more than 4 digits. Add decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places.	Introduce decimal place value counters and model exchange for addition. 		Then move onto decimals with a different number of decimal places.  
Y6 Add several numbers of increasing complexity. Including adding money, measure, and decimals with different numbers of decimals points.			

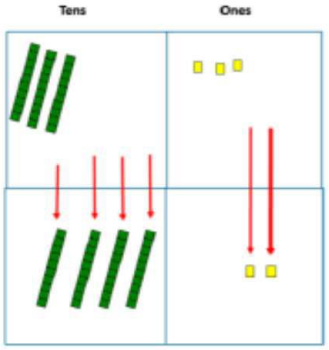
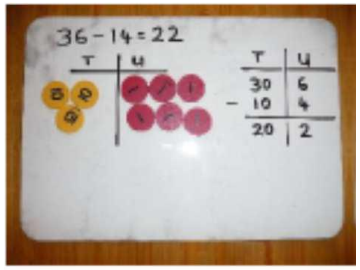
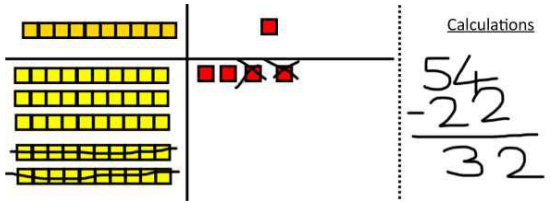
			 23.361 + 9.080 59.770 92.211 Insert 0 for the place holder if necessary.

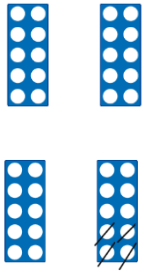
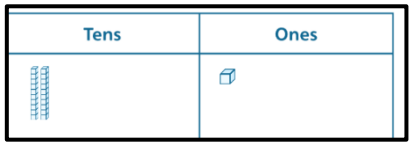
Year 1 Subtraction			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Taking away ones	Use real-life physical objects, counters, cubes etc. to show how objects can be taken away. 	Cross out drawn objects to show what has been taken away.  5 – 2 = 3 The bar model can also be used. 	4 = 6 – 2 18 - 3= 15 8 – 2 = 6   *Children need to see part whole model in different orientations.*
Counting back	Make the larger number in the subtraction calculation. Move the beads along the bead string whilst counting backwards in ones.	Count back in ones using a number line. 	Put 13 in your head, count back 4. What number are you at? Use your fingers to help. *Children will need regular practice counting backwards.*

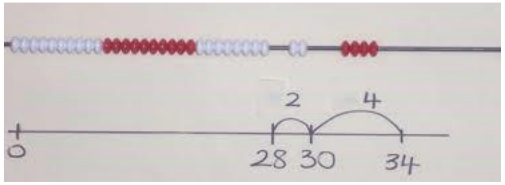
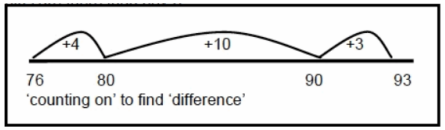
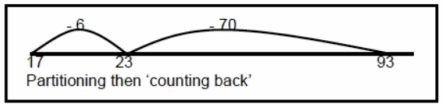
	 using number lines or number tracks, children start with 6 and count back 2.  Move objects away from the group, counting backwards. 	Children to represent what they see  pictorially e.g. Use the bar  Part + Part = Whole Whole – Part = Part	
Find the Difference	Compare amounts and objects to find the difference. Use cubes to build towers or make bars to find the difference.	Count on using a number line to find the difference  	Find the difference between 8 and 5. 8 – 5, the difference is ☐ Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.

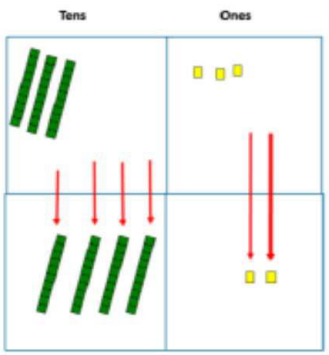
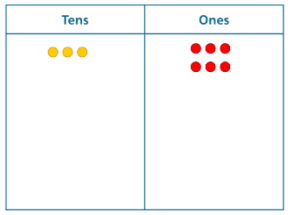
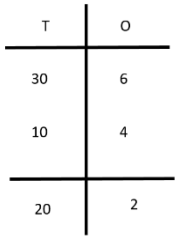
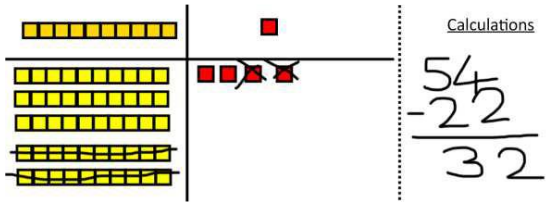
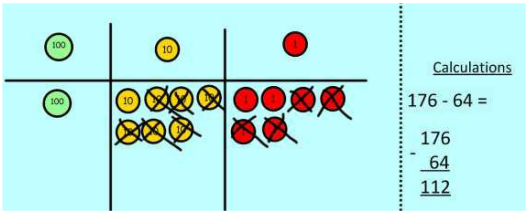
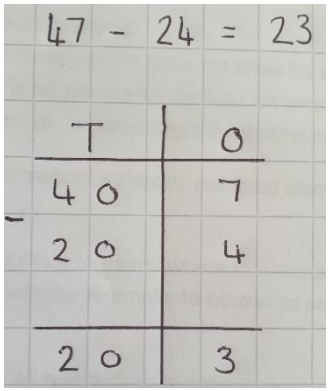
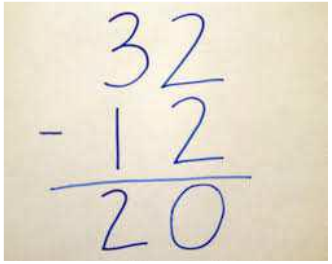
	  Use basic bar models with items to find the difference.  	**Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.*  Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.	Sarah has 8 sweets. Lara has 5 sweets. How many more does Sarah have?

			
Represent and use number bonds and related subtraction facts within 20 Part-part-whole model	If 10 is the whole and 5 is one of the parts. What is the other part? 10 - 5 = or 10 - ? = 5 	 Use pictorial representations to show the part.	Move to using numbers with the part-part-whole model.  10 - 5 = 5 or 5 = 10 - ?
Make 10	using ten frames. 14 – 5 	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. *This requires the children being secure in portioning numbers and recognising numbers within numbers.*	16 – 8 = How many do we take off to reach the next 10? How many do we have left to take off?

	Make 14 on the ten frame. Take away the 4 first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.		
Year 2 Subtraction			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Column method without regrouping	75- 42 =  Use Dienes to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first.  	Draw the Dienes or place value counters alongside the written calculation to help to show working.  	
Regroup a ten into ten ones.	20-4 =	20 -4 =	20—4 = 16

This step allows children the opportunity to exchange a ten for 10 ones, working with small numbers and understanding the relationship between the tens and ones.	 Start with 2 tens  exchange 1 ten for 10 ones.  take 4 ones away. You are left with 1 ten and 6 ones Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'		
Partitioning to subtract without re-grouping. *'Friendly numbers'*	34—13 = 21 Use Dienes to show how to partition the number when subtracting without regrouping. 34 becomes 3 tens and 4 ones.  Then subtract the 13, always reinforcing we start with the ones column.  34 - 13 Model a column method alongside to show the children the relationship *(children should only record in the way when ready)*	Children draw representations of Dienes and cross off.  43—21 = 22	43—21 = 22

Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	34—28  Use a bead bar or bead strings to model counting to next ten and the rest.	Use a number line to count on to next ten and then the rest. Use a number line to partition and then count back.   76 + 4 =80 80 + 13 = 93 so 93 – 76 = 4 + 13 = 17 'counting on' 93 - 76 = 93 - 70 - 6 93 - 70 = 23 23 - 6 = 17 'counting back'	93—76 = 17

Year 3 Subtraction			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Column method without regrouping (friendly numbers)	75- 42 =  Use Dienes to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first using PV counters.   Physically remove the 4 ones and then the 1 ten. Leaving 2 tens and 2 ones = 22.	Draw the Dienes or place value counters alongside the written calculation to help to show working.   	This will lead to a clear written column subtraction.  

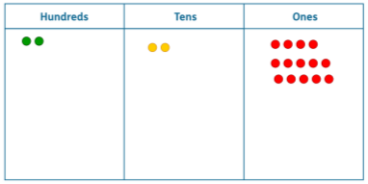
Column method **with regrouping**

*This builds on the step in YR2 where they **Regroup a ten into ten ones.***

Make the larger number with the Dienes or place value counters.

$$\begin{array}{r} 234 \\ -\ 88 \\ \hline \end{array}$$

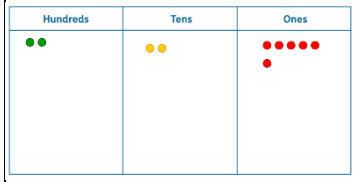
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



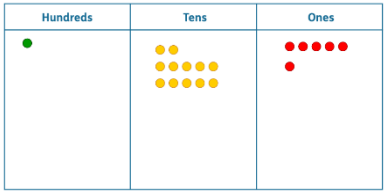
$$\begin{array}{r} 234 \\ -\ 88 \\ \hline \end{array}$$

Now I can subtract my ones.

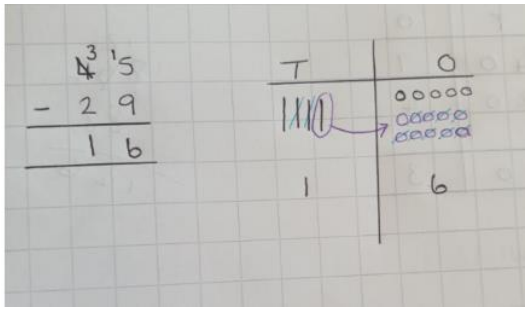
Now look at the tens, can I take away 8 tens easily?



I need to exchange one hundred for ten tens.

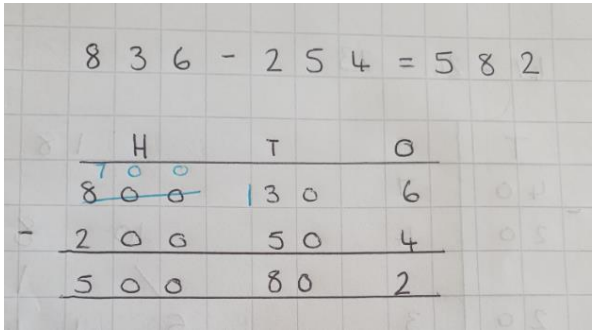


Children may draw base ten or PV counters and cross off.

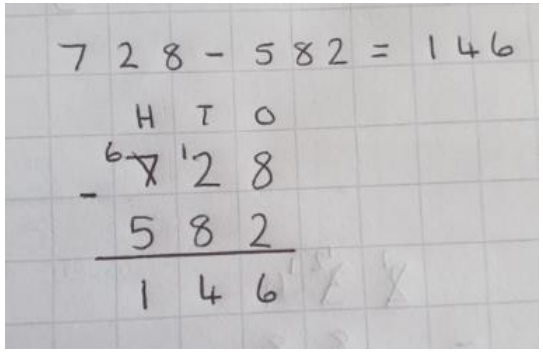


When confident, children can find their own way to record the exchange/regrouping.

Children can start their formal written method by partitioning the number into clear place value columns.

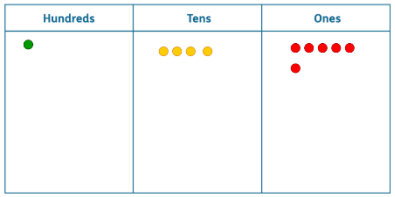
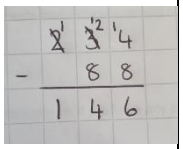
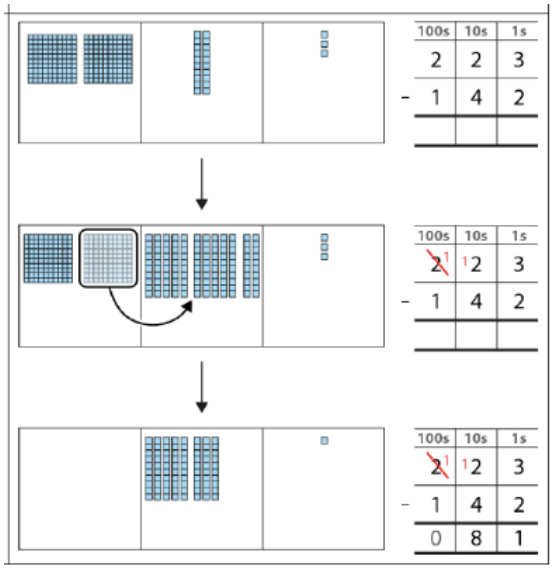


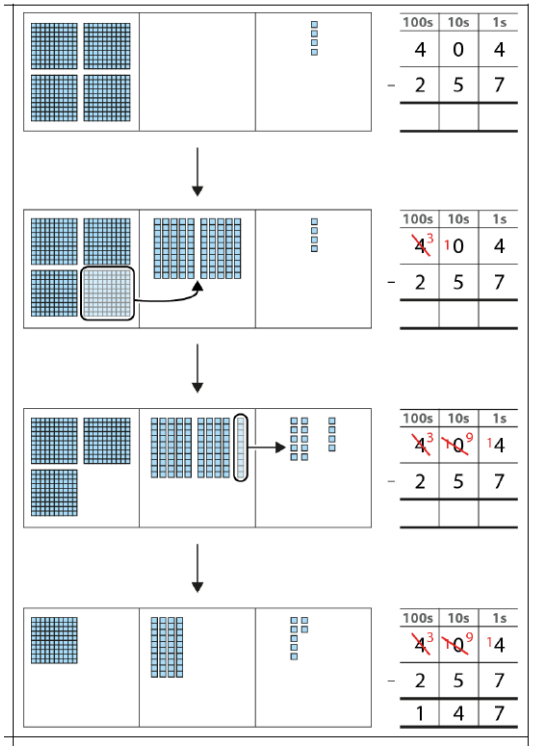
Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals.



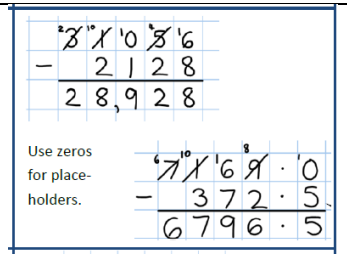
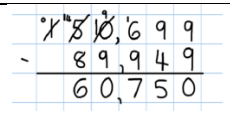
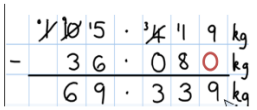
Column subtraction can be used when adding and subtracting money.

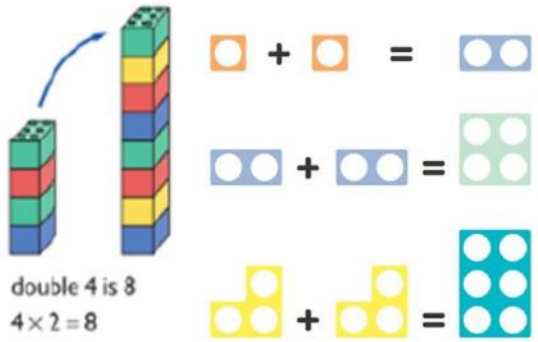
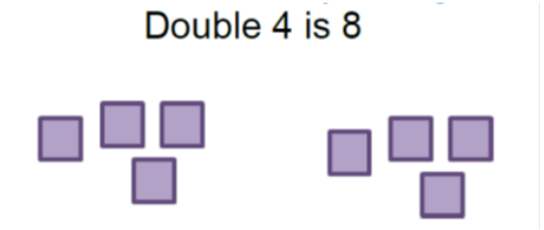
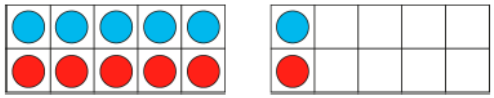
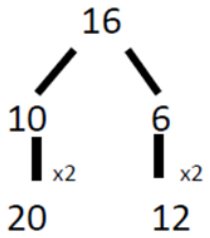
Remember to give children exposure to subtracting 2 digit numbers from 3 digit numbers.

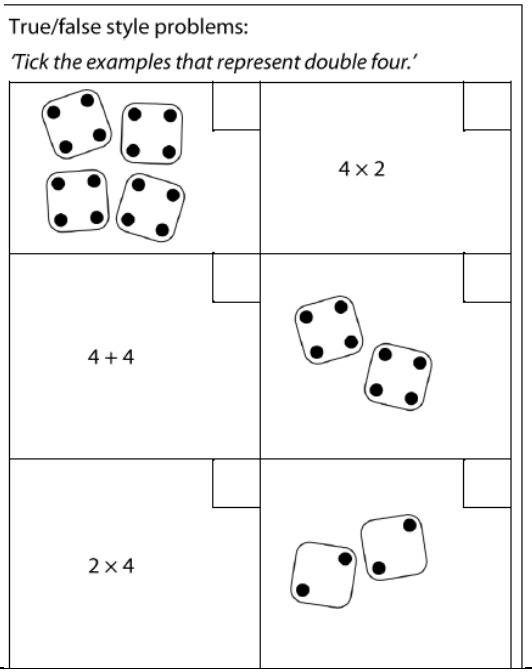
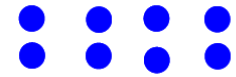
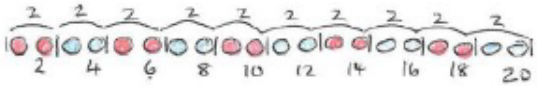
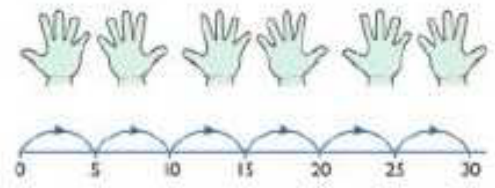
	Now I can take away eight tens and complete my subtraction.   Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.  Children need plenty of exposure to calculations which involve working through zero e.g. 404-257		5 6 3 − 2 1 3 4 8 2 − 1 9 7 8 2 4 − 3 1 9 4 0 5 − 1 2 3 3 1 6 − 1 0 3 9 0 3 − 1 2 4 To promote depth, present children with a range of calculations and ask questions such as: • *Which calculations require exchange?* • *Which calculations require exchange only once?* • *Which calculations require exchange twice?* • *Which calculations require exchange through zero?* Encourage children to look at the numbers involved and justify their answers in that way rather than just performing the calculations.

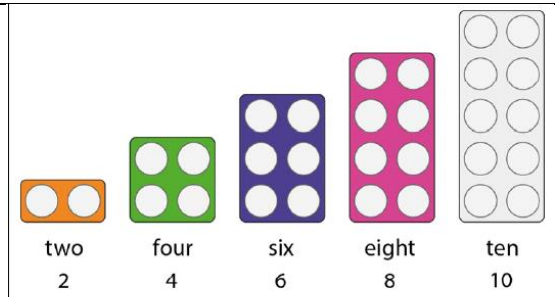

100s 10s 1s
4 0 4
− 2 5 7
100s 10s 1s
4 10 4
− 2 5 7
100s 10s 1s
4 0 14
− 2 5 7
100s 10s 1s
4 0 14
− 2 5 7
1 4 7

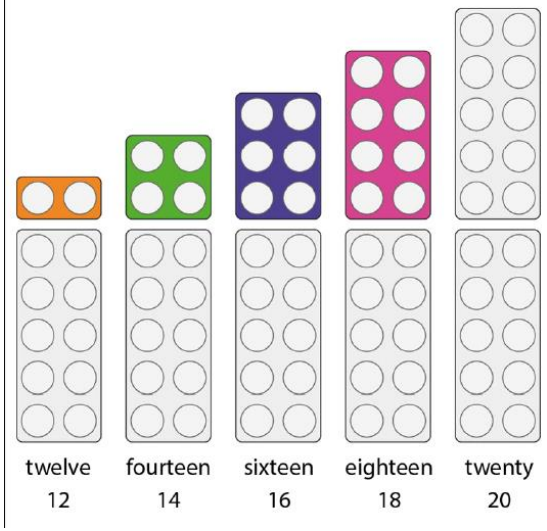
Year 4-6 Subtraction			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
	Building on steps earlier if needed.		
subtract with up to 4 digits. *Introduce decimal subtraction through context of money*		Column subtraction: • *'I have saved £6.53 and my brother has saved £4.38. How much more money have I saved than my brother?'* £6.53 | £4.38 | ?	*As above for depth* Column subtraction: • *'I have saved £6.53 and my brother has saved £4.38. How much more money have I saved than my brother?'* £ 6 . 5 4 1 3 – £ 4 . 3 8 £ 2 . 1 5 It is useful for the children to see a bar model alongside written methods. *'The class has raised £100 to spend on a party. They spend £25.49 on pizzas, £13.85 on drinks and £18.75 on decorations. How much do they have left to spend on the entertainment?'* • Step 1 £100 | £25.49 | £13.85 | £18.75 | ? £ 2 5 . 4 9 £ 1 3 . 8 5 + £ 1 8 . 7 5 **£ 5 8 . 0 9** 1 2 1 • Step 2 £100 | £58.09 | ? +1p +90p +£1 +£40 £58.09 £58.10 £59.00 £60.00 £100.00 £40 + £1 + 90p + 1p = £41 + 91p = £41.91 The class has £41.91 left to spend on entertainment.

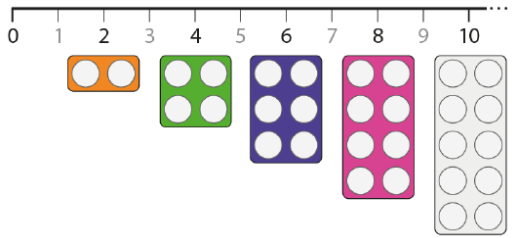
Yr5 - Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal			 31056 - 2128 = 7169 - 372.5 =
Yr6 - Pupils practise subtraction for larger numbers, using the formal written methods of columnar			 150699 - 89949 =  1105.419kg -36.08kg = *Children can use zeros for place holders.*

Year 1 Multiplication			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling  	Draw pictures to show how to double numbers  Doubling strategy for six to ten – example 1: • Tens frames and counters: 	Partition a number and then double each part before recombining it back together.  Completing multiplication equations: *'Fill in the missing numbers.'* 2 × 7 = ☐ 7 × 2 = ☐ Double 7 is equal to ☐ 7, twice is equal to ☐ We can write twice 6 as ☐ × 6 double 6 = ☐ We can write double 3 as 2 × ☐ this is the same as ☐ × 2 double 3 = ☐ 4 + 4 + 4 = 12

			True/false style problems: *'Tick the examples that represent double four.'* 
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting.  	Children make representations to show counting in multiples.    	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30

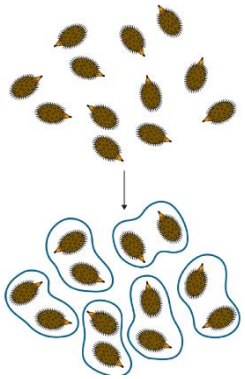


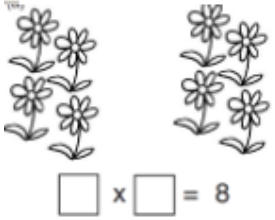
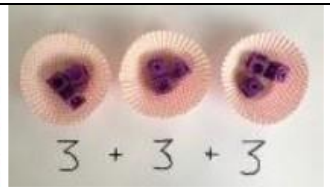
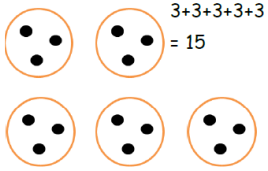
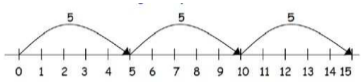


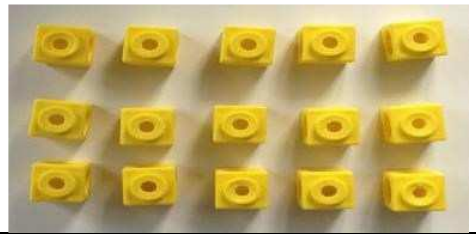
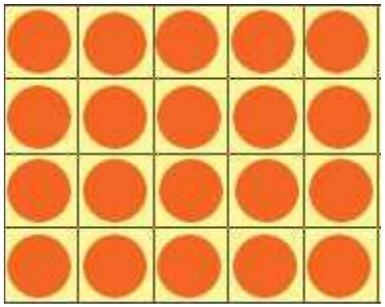
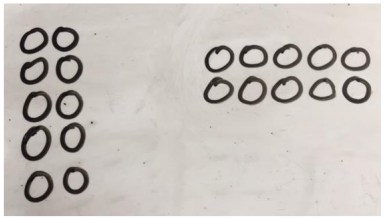


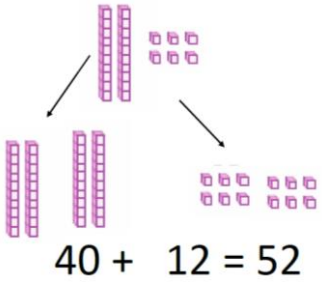
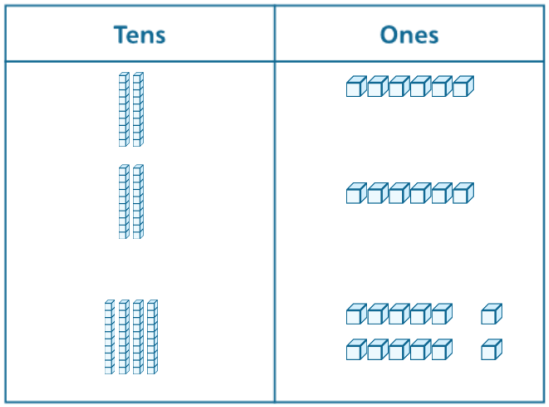
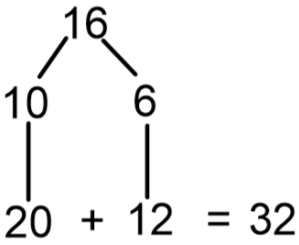
How many wheels are there? Count in group of 2.

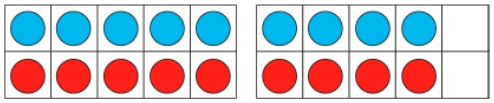
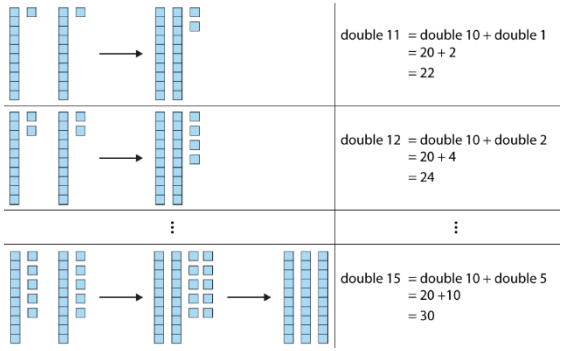
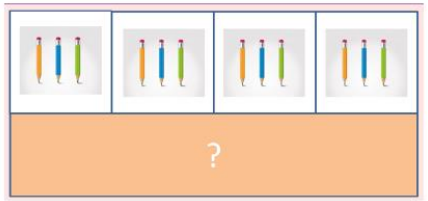
Once children can confidently 'pair off' concrete resources, progress to counting even quantities that are represented pictorially. Encourage children to draw rings around each group of two to support their counting.

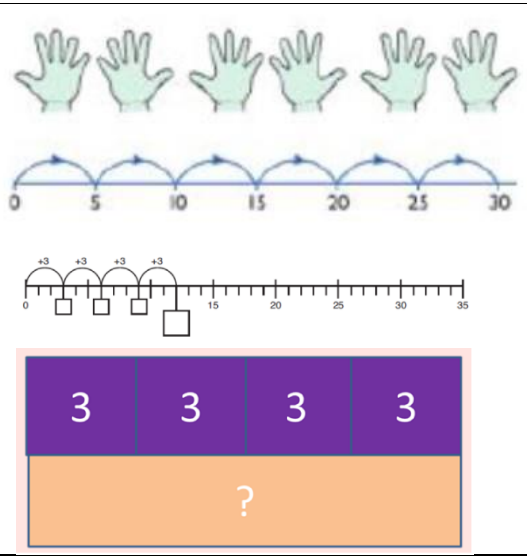
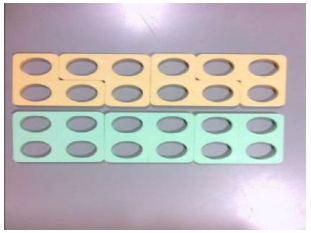
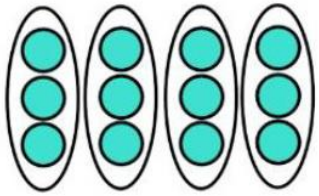
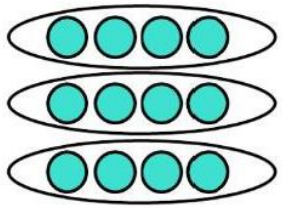


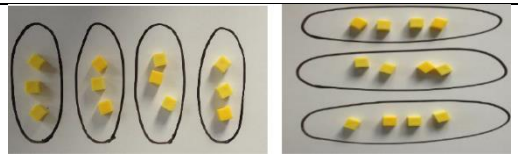
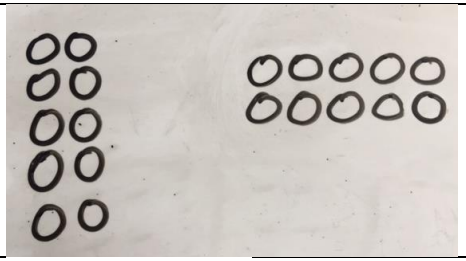
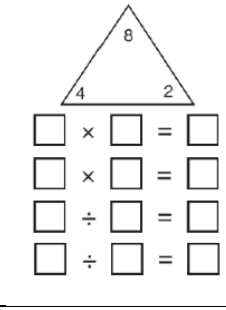
Making equal groups and counting the total	 	Draw  to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8
Repeated addition	  Use different objects to add equal groups	There are 3 sweets in one bag. How many sweets are in 5 bags altogether?  There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures. 

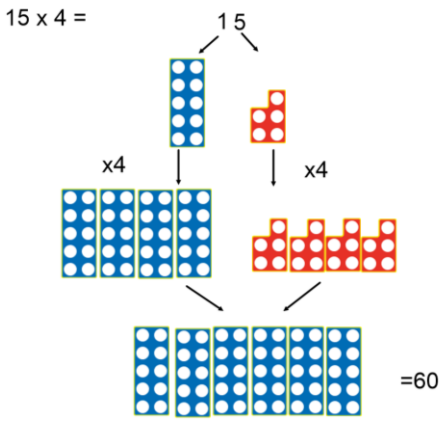
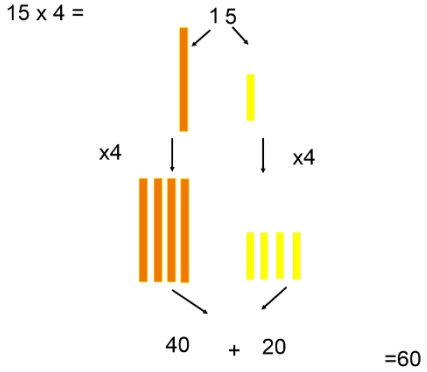
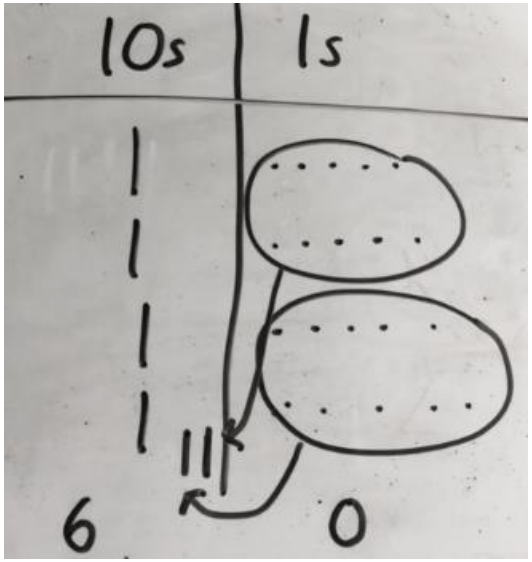
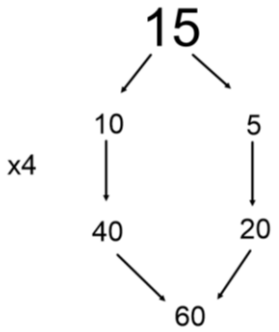
Understanding arrays	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.  	Children to be able to use an array to write a range of calculations e.g. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5

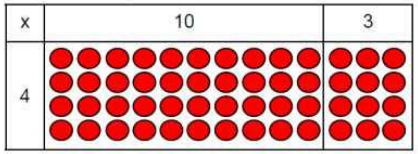
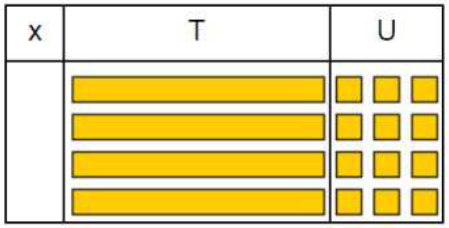
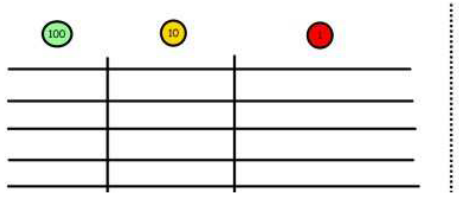
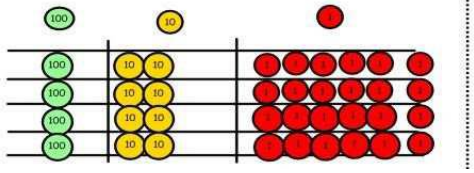
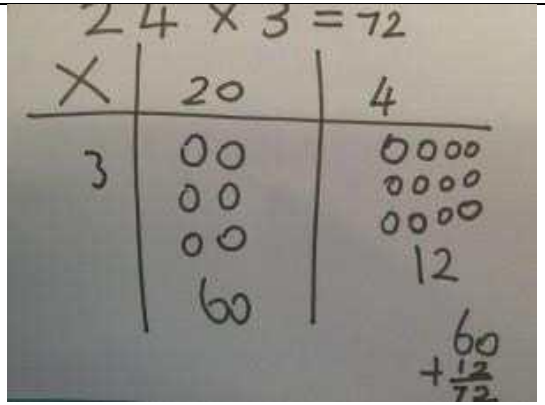
Year 2 Multiplication			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Doubling	Model doubling using dienes and PV counters. Double 26 - partition tens and ones.  40 + 12 = 52 Use ten frames and different coloured counters.	Draw pictures and representations to show how to double numbers. 	Partition a number and then double each part before recombining it back together. 

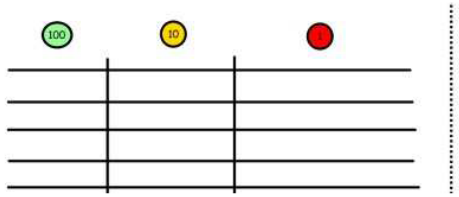
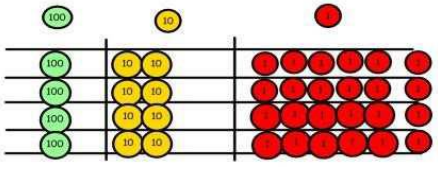
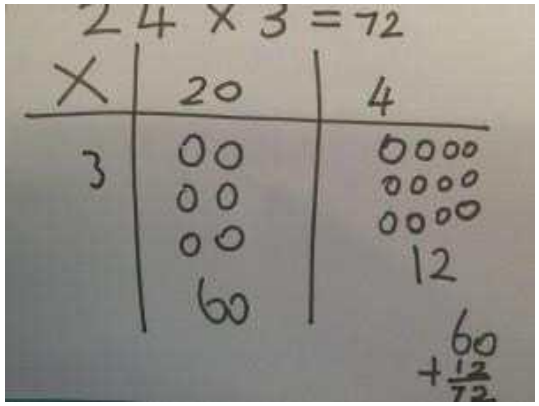
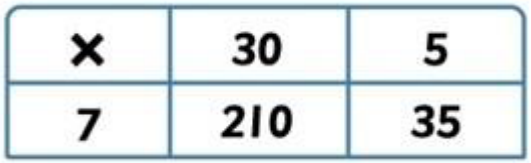
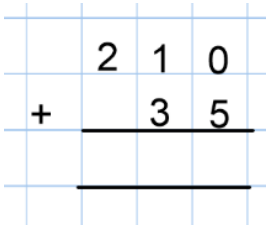
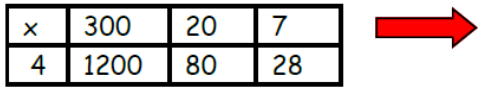
	 *'Nine is five plus four, so double nine is double five plus double four.'* 5 + 5 = 10 4 + 4 = 8 10 + 8 = 18 so 9 + 9 = 18 *'Nine is ten minus one, so double nine is double ten minus double one.'* 10 + 10 = 20 1 + 1 = 2 20 – 2 = 18 so 9 + 9 = 18 Use dienes to model doubling 		
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40 	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$

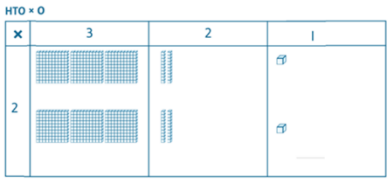
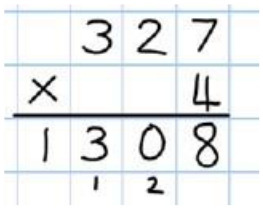
			
Multiplication is commutative	Create arrays using counters and cubes and Numicon.   Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	Use representations of arrays to show different calculations and explore commutativity.   Children to represent the arrays pictorially.	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

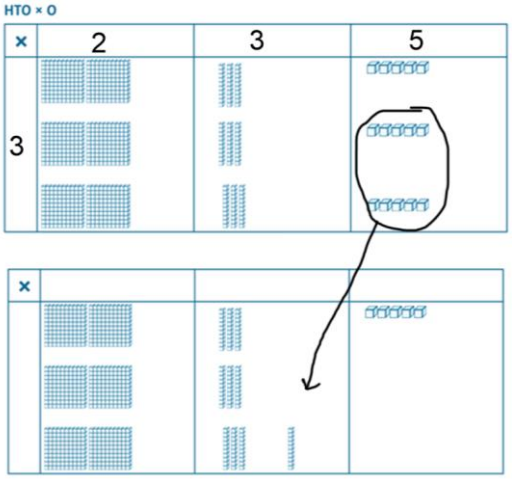
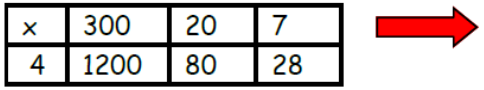
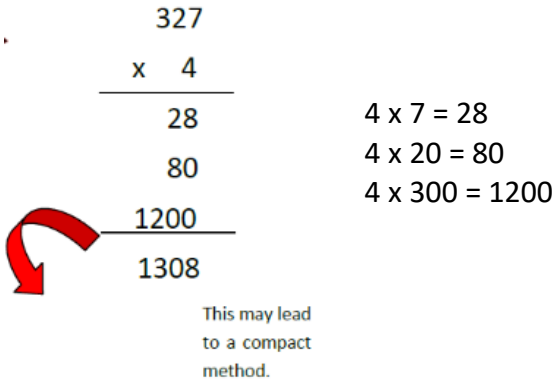
			
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.

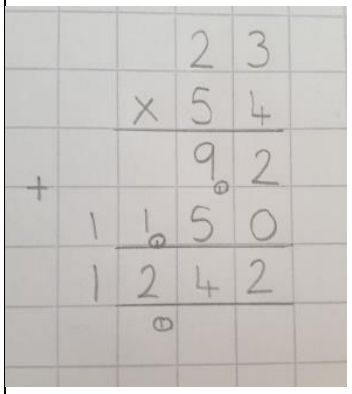
Year 3 Multiplication			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
TO x O Partitioning method Can also be used for HTO X O	Partition to multiply using Numicon, base 10 or Cuisenaire rods.  	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken.  10 x 4 = 40 5 x 4 = 20 40+ 20 = 60
Grid method	Show the links with arrays to first introduce the grid method.	Children can represent the work they have done with place value counters in a way that they understand.	Children to record what it is they are doing to show understanding.

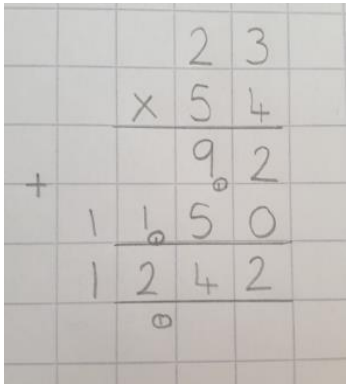
	 4 rows of 10 4 rows of 3	They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.	Begin by partitioning the number with the greatest number of digits down the left hand side so that partial products can be easily added
	Use Dienes to move towards a more compact method.  4 rows of 13 Use place value counters to show finding groups of a number eg. multiplying by 4 so we need 4 rows.  Calculations 4 x 126 Fill each row with 126  Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed.		8 40 320 7 56 376

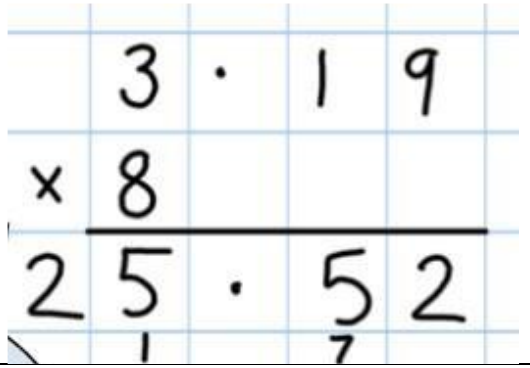
Year 4 Multiplication			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Recap Grid method from Yr3 Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)	Use place value counters to show finding groups of a number eg. multiplying by 4 so we need 4 rows.  Calculations 4 x 126 Fill each row with 126  Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed.	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.  	Children to record what it is they are doing to show understanding. Start with multiplying by one-digit numbers and showing the clear addition alongside the grid.  
Column multiplication	Children can continue to be supported by place value counters or Dienes at the stage of multiplication. This initially done where there is no regrouping before moving onto exchanged being made.	The grid method may be used to show how this relates to a formal written method.  	If it helps, children can write out what they are solving next to their answer.

	321 x 2 =  600 40 2 321 x 2 = 642 It is important at this stage that they always multiply the ones first. Always model the pictorial alongside the concrete for children to see the links in using manipulatives and recording.		327 x 4 28 80 1200 1308 This may lead to a compact method.  4 x 7 = 28 4 x 20 = 80 4 x 300 = 1200

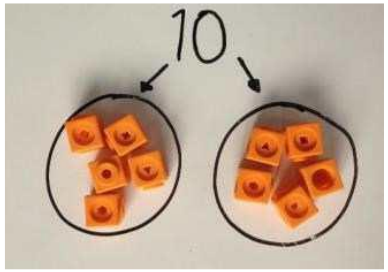
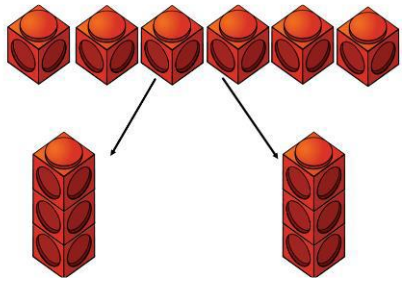
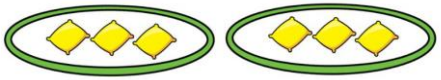
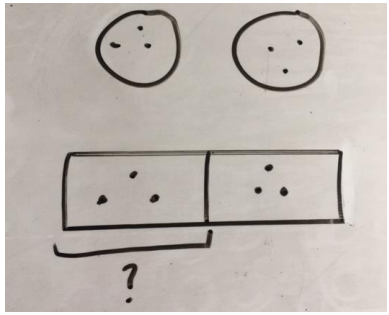
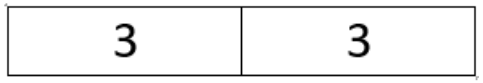
Year 5 Multiplication			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers			
Recap column multiplication from Yr4. Extend to 4 digit x 1			

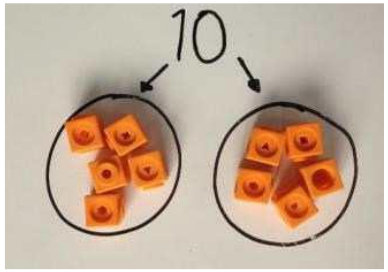
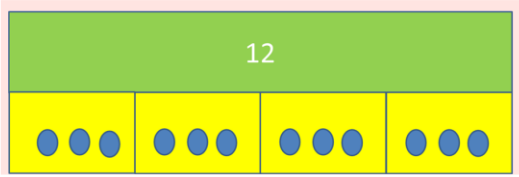
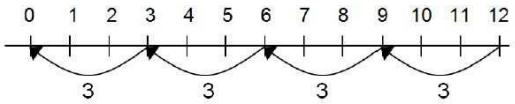
column multiplication **up to 4 digit** number by 2 digit number. Begin introducing 2 digit x 2 digit and then build up to up to 4 digitis.	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	23 x 54 = TO × TO × | 20 | 3 50 | 1000 | 150 4 | 80 | 12 1000 150 + 80 12 Reinforce some calculations can be calculated mentally.	327 x 53 981 ← 327 x 3 16350 ← 327 x 50 17331 When modelling long multiplication use the following language and layout.  •4 multiplied by 3 is 12. Write the 2 in the ones column and carry one ten. •4 multiplied by 2 is 8. Now add on the carrying figure (1) giving 9 tens altogether. (At this stage of development children will know that we are not actually multiplying 4 by 2 but 4 by 20, or 2 tens but reinforce this concept) Write the answer in the tens column •Now multiply by the tens column. As we are multiplying by 10 place a zero in the ones column. Ensure children know why we do this. •5 multiplied by 3 is 15. Place the 5 in the tens column and write the carrying figure •5 multiplied by 2 is 10. Now add the carry figure (1) giving a total of 11. •Now add all the partial products using the columnar method for addition.

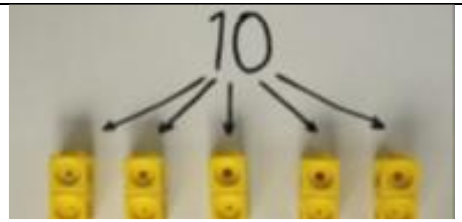
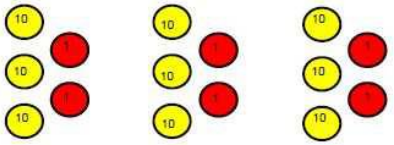
Year 6 Multiplication			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers			
column multiplication **up to 4 digit** number by 2 digit number. Build on Yr5.	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	23 x 54 = TO × TO × | 20 | 3 50 | 1000 | 150 4 | 80 | 12 1000 150 + 80 12 Reinforce some calculations can be calculated mentally.	When modelling long multiplication use the following language and layout.  •4 multiplied by 3 is 12. Write the 2 in the ones column and carry one ten. •4 multiplied by 2 is 8. Now add on the carrying figure (1) giving 9 tens altogether. (At this stage of development children will know that we are not actually multiplying 4 by 2 but 4 by 20, or 2 tens but reinforce this concept) Write the answer in the tens column •Now multiply by the tens column. As we are multiplying by 10 place a zero in the ones column. Ensure children know why we do this. •5 multiplied by 3 is 15. Place the 5 in the tens column and write the carrying figure •5 multiplied by 2 is 10. Now add the carry figure (1) giving a total of 11. •Now add all the partial products using the columnar method for addition.
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the ones column. Line up the decimal points in the question and the answer.

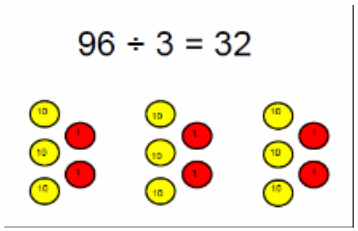
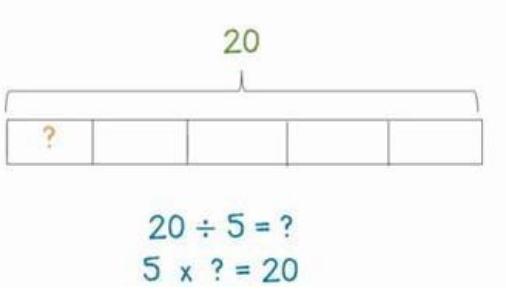
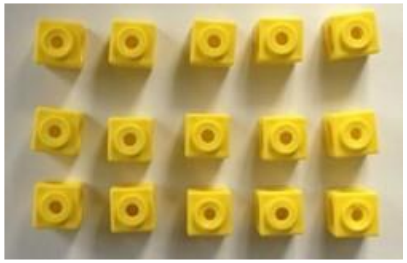
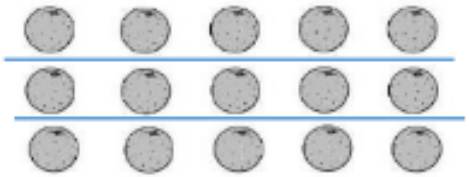
			

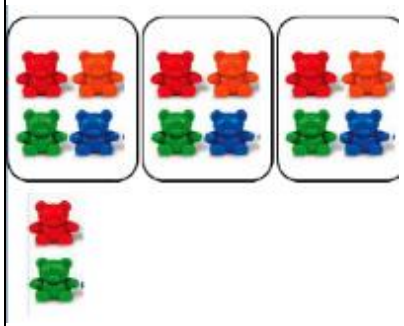
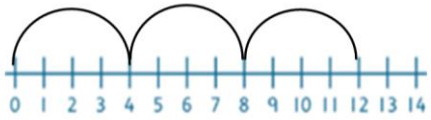
Year 1 Division			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**

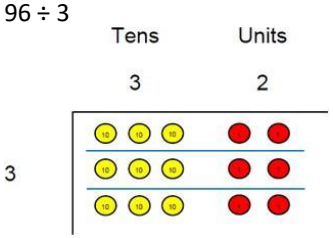
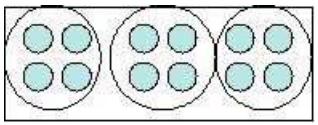
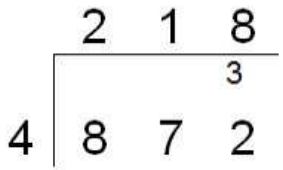
Division as sharing Sharing using a range of objects.	 I have 10 cubes; can you share them equally into 2 groups.  	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4  	Children should also be encouraged to use their 2 times tables facts. 6 ÷ 2 = 3 

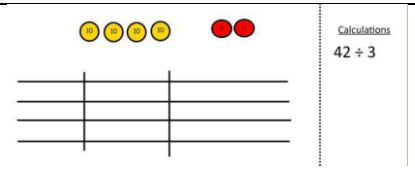
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing Sharing using a range of objects. *Sharing - the quotient represents the quantity of shared objects in each group*	 I have 10 cubes; can you share them equally into 2 groups.	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 Children use bar modelling to show and support understanding. 	12 ÷ 3 = 4
Division as grouping *When grouping, the quotient represents the amount of groups within the shared quantity*.	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Use a number line to show jumps in groups. The number of jumps equals the number of groups. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

	  96 ÷ 3 = 32 		

Year 3 Division			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding. 24 divided into groups of 6 = 4  	Continue to use bar modelling to aid solving division problems. 	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created.	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 It's important for children to continue to be exposed to the = at the start of the calculation. 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

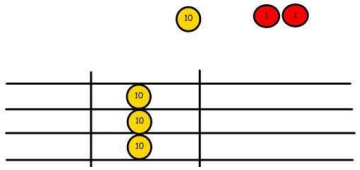
	15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15		
Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 14 ÷ 3 =  Draw dots and group them to divide an amount and clearly show a remainder.  	Complete written divisions and show the remainder using r. 

Year 4-6 Division			
Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Divide at least 3 digit numbers by 1 digit. Short Division	96 ÷ 3  Use place value counters to divide using the bus stop method alongside.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently. Children can represent using base 10 pictorially.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.

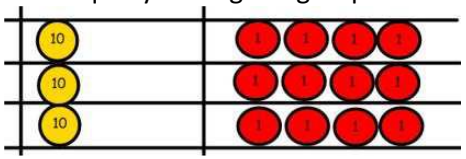


42 ÷ 3=

Start with the biggest place value; share 40 into three groups. Put 1 ten in each group then 1 ten left over.

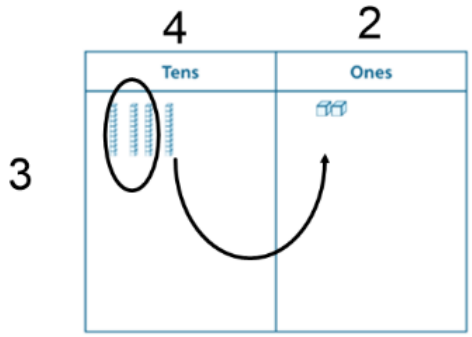


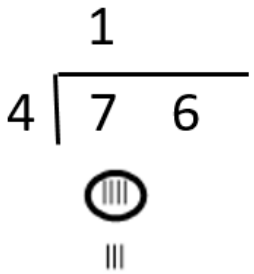
Exchange this ten for ten ones and then share the ones equally among the groups.

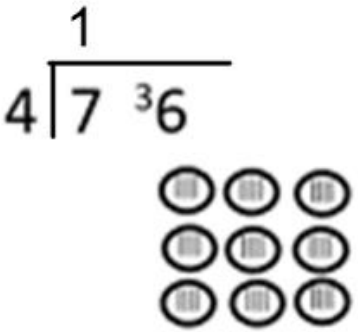


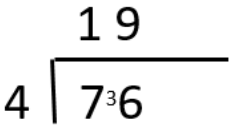
Look how much is in 1 group so the answer is 14.

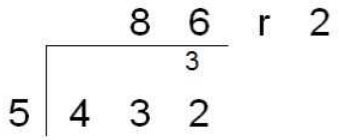
Set out the calculation as shown below.
Put the ten rods into groups of 3. How many groups can you make? 1





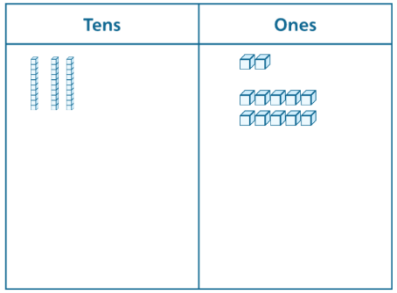






Children need to be able to deal with remainders accurately, in context and in a variety of ways (as a remainder, as a fraction and as a decimal)

Therefore 19 R 2 should also be explained as 19 2/4 (a '2 out of 4' group) and as 19.5

	Now exchange the 1 remaining ten rod for 10 one cubes. Put the one cubes into groups of 3. How many groups can you make?  		
Long Division			e.g. 432 ÷ 15 $$\begin{array}{r} 28 \\ 15\overline{)432} \\ 30 \\ \hline 132 \\ 120 \\ \hline 12 \end{array}$$ As this method is very abstract, it should not be introduced until children are ready (year 6.)

			• "How many 15s are there in 4?" There are none so now ask "how many 15s are there in 43?" • There are 2. Write 2 on the answer line above the 3 and write 30 (15x2) beneath the 43. • Subtract 30 from 43 and write the answer (13) beneath the 30. • Check that your intermediate answer is smaller than the divisor. If the answer is larger than the divisor (e.g. more than 15) then go back and start again as the first division is incorrect. • Now bring down the next figure (2) and place this digit on the end of the intermediate answer. (e.g. 132) • "How many 15s are there in 132?" There are 8 (15x8 = 120) Write the 8 on the answer line and write 120 (15x8) beneath 132. • Subtract 120 from 132 and write the answer (12) beneath the 120. • As this answer is smaller than the divisor of 15 then this is the remainder. (28 R 12) **Children need to be able to deal with remainders accurately, in context and in a variety of ways (as a remainder, as a fraction and as a decimal) Therefore 28 R 12 should also be explained as 28 12/15 (a '12 out of 15' group which can be simplified to 4/5). As a decimal this can be explained as 28.8** **<u>Note</u>**

			It's often helpful it children write down the first few multiples of the divisor e.g. X 1 – 15 X 2 – 30 X 3 – 45 X 4 – 60 X 5 – 75 They can then use these multiples to generate others quickly if needed. E.g. 15 x 7 = (15 x3 (45) + 15 x 4(60) = 105